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Charge Independence and Symmetry of Nuclear Forces

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Abstract:

Charge independence and symmetry are approximate symmetries of nature. The observations of the small symmetry breaking effects and the consequences of those effects are reviewed. The effects of the mass difference between up and down quarks and the off shell dependence q^2 of ρ^0-ω mixing are stressed. In particular, I argue that models which predict a strong q^2 dependence of ρ^0-ω mixing seem also to predict a strong q^2 variation for the ρ^0-γ^* matrix element, in contradiction with experiment.

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1. Introduction

The topic of this paper is one of the many on which Ernest Henley has made seminal contributions. I have been privileged to work with him and he has taught me a great deal.

The outline is as follows. I shall begin by defining the terms charge independence and charge symmetry. Charge independence breaking of the $^1S_0$ nucleon-nucleon scattering lengths is discussed briefly. The subject has been pretty well explained since the 1966 paper of Henley & Morrison. The bulk of the remainder is concerned with the breaking of charge symmetry (CSB). I shall review the evidence that the positive value of the light quark mass difference m_d-m_u plus electromagnetic effects accounts for CSB in systems of baryon number ranging from 0 to 208.

2. Definitions

In the limit that m_d and m_u vanish and, ignoring electromagnetic effects the u and d quarks are equivalent. They form an isodoublet $\binom{u}{d}$. One may introduce the isospin operators $\vec{\tau}$ with $[\tau_i, \tau_j] = i\ \epsilon_{ijk}\tau_k, \tau_3|u>=|u>$ and $\tau_3|d>=-|d>$. The total isospin for a system of quarks is then $\vec{T} = \Sigma\vec{\tau}(i)/2$. In the limit in which each of m_d, m_u, α vanishes $[H, \vec{T}] = 0$. This vanishing, equivalent to the invariance under any rotation in isospin space is called charge independence. Charge symmetry requires only an invariance about rotations by π about the α axis in isospin space: $[H, P_{cs}] = 0$, with $P_{cs} = e^{i\pi T_2}$. P_{cs} converts u quarks into d quarks and vice versa: $P_{cs}|u>=-|d>$, $P_{cs}|d>=|u>$.

Henley's 1969 review explained why it is important to distinguish between charge independence and charge symmetry.

There is a legitimate concern about the application of these concepts to reality. While each of m_d and m_u is less than 10 MeV, it is well known that $\frac{m_d}{m_u} \approx 2$; see the review [3]. Thus one may wonder why any trace of charge independence would remain in nature. However the strong interaction effects of confinement cause the ratios governing charge independence breaking to be $\sim \frac{m_d-m_u}{300\text{ MeV}}$ or $\frac{m_d-m_u}{\Lambda_{QCD}}$ or $\frac{m_d-m_u}{4\pi f_\pi}$. The ~ 300 MeV can be thought of as arising from a constituent quark mass, bag model energy or quark conden-

sate. Thus the effects of $m_d - m_u > 0$ are small, as are the electromagnetic effects. Thus charge independence holds approximately. This is well known, as hadronic and nuclear states are organized as isomultiplets.

The symmetry is not perfect and gives a unique opportunity to search for clues about the underlying dynamics. A prominent example is that the positive value of $m_d - m_u$ causes the neutron to be heavier than the proton.

3. Nucleon-Nucleon Scattering - $^1S_0$ Channel

Charge independence $[H, \vec{T}] = 0$ imposes the equalities of the nucleon-nucleon scattering lengths $a_{pp} = a_{nn} = a_{np}$. But electromagnetic effects are large and it is necessary to make corrections. The results are analyzed, tabulated and discussed in Ref. 4. These are

$$\begin{aligned} a_{pp} &= -17.7 \pm 0.4 \text{ fm} \\ a_{nn} &= -18.8 \pm 0.3 \text{ fm} \\ a_{np} &= -23.75 \pm 0.09 \text{ fm} \end{aligned} \tag{1}$$

The differences between these scattering lengths represent CIB and CSB effects. There are very large percentage differences between these numbers which may seem surprising. But one must recall that that is the inverse of the scattering lengths that are related to the potentials. For two different potentials, V_1, V_2 the scattering lengths a_1, a_2 are related by

$$\frac{1}{a_1} - \frac{1}{a_2} = M \int dr\; u_1(V_1 - V_2)u_2 \tag{2}$$

where u_1 and u_2 are the wave functions. The differences between the inverse of the scattering lengths are small and furthermore [2]

$$\frac{\Delta a}{a} = (10 - 15)\frac{\Delta V}{V}. \tag{3}$$

One defines Δa_{CD} to measure the CIB, with

$$\Delta a_{CD} = \frac{1}{2}(a_{pp} + a_{nn}) - a_{np} = 5.7 \pm 0.3 \text{ fm}. \tag{4}$$

This corresponds to a charge dependence breaking of about 2.5% [2]. The violation of charge symmetry is represented by the quantity

$$\Delta a_{CSB} = a_{pp} - a_{nn} = 1.5 \pm 0.5 \text{ fm}. \tag{5}$$

It is natural to use meson exchange models to analyze these low energy data. The longest range force arises from the one pion exchange potential OPEP, which also supplies significant breaking of charge independence. This is due to the relatively large mass difference. $\frac{m_{\pi^\pm}-m_{\pi^0}}{m_{\pi^0}} \approx 0.04$. One might worry about including the charge dependence of the coupling constants for neutral (g_0) and charged (g_c) pions. However $g_0^2 = g_c^2$ to better than about 1%, according to recent phase shift analyses of Bugg and Machleidt [6] and the Nijmegen [7] groups. One must also include the effects of the π mass difference in the two pion exchange potential TPEP. Henley & Morrison were the first to do that.

Some computations [2,7,8] of Δa_{CD} are displayed in Table 1. One can see that the agreement with the experimental value of $\Delta a_{CD} = 5.7 \pm 0.5$ fm is very good. There is room for a small contribution from quark effects. The net result is that the understanding of charge dependence has been rather good.

4. Charge Sysmmetry Breaking - $\rho^0\omega$ Mixing

The strongest and most prominent observation of charge symmetry breaking occurs in $\rho^0\omega$ mixing. The wave functions are given schematically as

$$\begin{aligned} |\rho^0> &= \frac{1}{\sqrt{2}}\Big(|u\bar{u}> - |d\bar{d}>\Big) \\ |\omega> &= \Big(|u\bar{u}> + |d\bar{d}>\Big), \end{aligned} \tag{6}$$

so that

$$< \rho^0|H|\omega >= \frac{1}{2} < u\bar{u}|H|u\bar{u}> - \frac{1}{2} < d\bar{d}|H|d\bar{d}> . \tag{7}$$

This vanishes unless the Hamiltonian includes effects that distinguish between the u and d quarks. One simple example is the mass terms which contribute $m_u - m_d$. Thus the mixing matrix element is strongly influenced by the quark mass difference. Electromagnetic effects also enter, as we shall discuss.

The effects of this matrix element are observed [11,12] in the annihilation process $e^+e^- \to \pi^+\pi^-$. The relevant diagrams are shown in Fig. 1 and the huge signal arising from the small widths of the ω-meson is displayed in Fig. 2. The mixing matrix element has been extracted [13] to be

$$< \rho^0|H|\omega > \approx -4500 \text{ MeV}^2. \tag{8}$$

This matrix element includes the effect of the electromagnetic process depicted in Fig. 3. The quantities f_ρ and f_ω have been determined from the processes $e^+e^- \to \rho, \omega \to e^+e^-$. The most recent analysis [14] gives $< \rho^0|H_{em}|\omega >= 640 \pm 140$ MeV^2 so that the strong contribution ($H = H_{str} + H_{em}$) is given by $< \rho^0|H_{str}|\omega > \approx -5100$ MeV^2. Another notable feature is that the electromagnetic contribution to the $\rho\omega$-mixing self-energy is of the form

$$\Pi^{em}_{\rho\omega}(q^2) \sim \frac{e^2}{f_\rho f_\omega} \frac{1}{q^2} \tag{9}$$

where q^2 is the square of the vector meson four-momentum.

It is natural to use the exchange of a mixed $\rho^0\omega$ meson as a mechanism for charge symmetry breaking nucleon-nucleon forces. This is shown in Figs. 4a and 4b. The electromagnetic contribution Fig. 4b is part of the long range, mainly Coulomb, electromagnetic interaction. The strong interaction term gives a nucleon-nucleon force of a medium range. This leads to a contribution to Δa_{CSB} of 1.4 fm, obtained by rescaling the Coon-Barrett [13] result by the ratio $1.11 = \left(\frac{5100}{4600}\right)$. This accounts for the observed effect $\Delta a_{CSB} = 1.5$ fm $\pm\, 0.5$ fm, while other effects seem small [13].

But this agreement with the experiment may not be satisfactory. A significant extrapolation is involved since $< \rho^0|H_{str}|\omega >$ is determined at $q^2 = m_\rho^2$, while in the NN force the relevant q^2 are spacelike, less than or equal to zero. Goldman, Henderson and Thomas [15] investigated the possible q^2 dependence of $< \rho^0|H_{str}|\omega >$ by evaluating the diagram of Fig. 6 using free quark propagators. They obtained a substantial q^2 dependence. The use of such a $< \rho^0|H_{str}|\omega >$ kills the resulting charge symmetry breaking potential. Very similar results were also obtained in the work of Refs. 16-19.

My opinion is that the charge symmetry breaking effects of the d-u mass difference in vector exchanges must persist, with little variation in q^2. However, I shall examine the consequences of the idea that $<\rho^0|H_{str}|\omega>$ does have a strong variation with q^2.

Consider the results of the "minimal" model of Krein, Thomas and Williams [17] which are displayed in Fig. 6. This work models confinement in terms of pole-less quark propagators. The rapid decrease of $<\rho^0|H_{str}|\omega>$ as q^2 is changed from time-like to space-like leads to a nearly vanishing CSB nucleon-nucleon interaction. But I stress that models which obtain the q^2 dependence of $<\rho|H_{str}|\omega>$ from the diagram Fig. 5 have an implicit prediction for the q^2 variation of the ρ-γ^* transition matrix element $e/f_\rho(q^2)$, see Fig. 7. My evaluation of this using the minimal model of Ref. [17] is shown in Fig. 8. A significant variation is seen, with a gain of a factor of 4 in the magnitude of $e/f_\rho(q^2)$. This is a noteworthy observation because $f_\rho(q^2)$ can be extracted from $e^+e^- \to \rho \to e^+e^-$ data at $q^2 = M_\rho^2$ and from the high energy $\gamma + P \to \rho^0 + \rho$ reaction at $q^2 = 0$. The results of many experiments are discussed in the beautiful review of Bauer, Spital, Yennie and Pipkin [20]. They summarize $f_\rho^2(q^2 = M_\rho^2)/4\pi = 2.11 \pm 0.06$ and $f_\rho^2(q^2 = 0)/4\pi = 2.18 \pm 0.22$, as obtained from experiments at the CEA, DESY, SLAC and Cornell. Real photon data at γ energies from 3 to 10 GeV are used in the analysis.

No variation of $f_\rho(q^2)$ with q^2 is found! This seems to be in strong disagreement with the consequences of the models of Refs. 15-19. The survival of such models seems to depend on finding a new way to account for the $\gamma + P \to \rho^0 + P$ data as well as for data on many γ-nucleon and nuclear reactions.

For this article I shall assume that $<\rho^0|H_{str}|\omega>$ has little dependence on q^2. Then charge symmetry breaking in the $^1\mathrm{S}_0$ channel is accounted for.

5. Charge Symmetry Breaking in the np System

Searches for charge symmetry breaking in neutron-proton scattering offer an opportunity to find CSB effects not present in the nn or pp system. These class IV forces of Henley and Miller [22] have the form

$$V^{IV} = (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L}(\tau_1 - \tau_2)_3 A + (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{L}(\vec{\tau}_1 \times \vec{\tau}_2)_3 B \tag{10}$$

where A and B are reasonable operators. The A term receives contributions from γ, and ρ^0-ω exchanges. B is dominated by π exchange effects. These operators cause the analyzing powers of polarized neutrons $A_n(\theta_n)$ and polarized protons to differ $A_p(\theta_p)$. Measurements [23,24] compare scattering with polarized neutron beam to neutron scattering on a polarized proton target. Time reversal invariance relates the latter measurement to A_p. These analyzing powers pass through zero at one angle θ_0 for the energy of TRIUMF [23] and IUCF [24] beams. If θ_0 for polarized neutrons differs from θ_0 obtained from polarized protons, then $\Delta\theta = \theta_0(n) - \theta_0(p) \neq 0$ and charge symmetry has been violated. Such observations were made in two beautiful experiments [23,24]. The results presented in terms of $\Delta A = \frac{dA}{d\theta}\Delta\theta$, are shown in Fig. 9. The calculations use the Bonn meson-exchange potential so that all of the parameters governing the strong interaction are pre-determined. (Other calculations are discussed in Ref. 4.) The agreement between theory and experiment is very good. A pion exchange effect arising from the presence of the n-p mass difference in the evaluation of the vertex function dominates the 477 MeV measurement. The ρ^0-ω mixing term has a significant but non-dominating influence at 183 MeV.

6. The ^{3}He-^{3}H Binding Energy Difference

The ground states of ^{3}H and ^{3}He would have the same binding energy if charge symmetry holds. Instead $B(^3\text{H}) - B(^3\text{He}) = 764$ keV. The neutron rich system is more deeply bound. The bulk of the difference is due to the Coulomb interaction and other electromagnetic effects. The determination of the strong charge symmetry breaking relies on the ability to make a precise evaluation of such effects. The three body system is the

best for such evaluations because the electromagnetic terms can be evaluated in a model independent way using measured electromagnetic form factors [27]. Coon & Barrett used recent Saclay data to obtain

$$\Delta B(em) = 693 \pm 19 \pm 5 \text{ keV}, \tag{11}$$

where the first uncertainty is due to the determination of the form factors, and the second to the small model dependence of some relativistic effects. Similar values of $\Delta B(em)$ were obtained in Ref. [28]. The difference between 764 and 693 is 71 keV, to be accounted for by charge symmetry breaking of the strong interaction. The use of $\rho^0\omega$ exchange potential which reproduces Δa_{CSB} yields about 90 $\pm$ 14 keV in good agreement. The errors allow some room for other small effects such as $\pi\eta$ or $\pi\gamma$ exchanges.

7. Nolen Schiffer Anomaly

The mirror nuclei (N, Z) and $(N-1, Z+1)$ have the same binding energy, if charge symmetry holds. Nolen and Schiffer made an extensive analysis of the electromagnetic effects which dominate the observed binding energy difference. After removing the electromagnetic effects the neutron rich nuclei were more deeply bound (by about 7%) than the proton rich nuclei. Including additional detailed nuclear structure effects reduced the number to about 5%, see the review [4]. A related problem occurs in understanding the energy difference between nuclei with $T > 1/2$ (^{48}Ca, ^{90}Zr, ^{208}Pb) and their isobaric analog states.

Blunden and Iqbal took up the challenge of seeing if a charge symmetry violating nucleon-nucleon potential, consistent with Δa_{CSB} could account for the missing 5% attraction. As shown in Table 2, it did. Actually I have rescaled the contributions due to $\rho^0\omega$ mixing to reflect my present value of $< \rho^0|H_{str}|\omega >= -5100$ MeV^2. The agreement is good but not perfect. Similar results have been obtained in Refs. [32] and [33].

The main point is that the anomaly is gone. CSB effects consistent with those observed in the NN system account for the bulk of the missing binding energy. There is some room for other effects such as nuclear-medium enhancements of the role of the d-u

quark mass difference due to scalar effects [33-36]. In any case the ultimate source of nuclear CSB is the light quark mass difference.

Note also that the use of CSB and CIB forces consistent with the NN data allows an explanation of the A dependence of non-Coulomb effects in the parent-analog mass differences [37]. The use of such forces is now a standard part of shell model calculations [38].

8. Summary

1. Charge independence breaking in the $^1S_0$ system is well explained [1,7,8].
2. Charge symmetry breaking is caused by the d-u quark mass difference $m_d - m_u > 0$, along with electromagnetic effects.
3. Measuring the e^+e^--$\pi^+\pi^-$ cross section at $q^2 \approx M_\omega^2$ allows an extraction of the strong contribution to the ρ-ω mixing matrix element $< \rho^0|H_{str}|\omega > \approx$ - 5100 MeV^2.
4. The TRIUMF (477 MeV) and IUCF (183 MeV) experiments compare analyzing powers of $\vec{n}p$ and $\vec{p}n$ scattering and observe CSB at the level expected from π, γ and ρ^0-ω exchange effects.
5. The ρ^0-ω exchange potential accounts for $\Delta a_{CSB} = a_{pp} - a_{nn} = 1.5 \pm 0.5$ fm.
6. The use of such a potential accounts for the strong CSB contribution to the ^{3}He-^{3}H mass difference.
7. The use of potentials consistent with Δa_{CSB} and Δa_{CIB} accounts for formerly anomalous binding energy differences in mirror nuclei and in analog states.

The quark mass difference seems to be related to a large variety of phenomena in particle and nuclear physics. Most of the effects are well understood. Perhaps the next relevant question is why are there two light quarks with a slightly different mass?

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Figure Captions

- Figure. 1. Amplitudes for $e^+e^- \to \pi^+\pi^-$
- Figure 2. $\sigma(e^+e^- \to \pi^+\pi^-)$. These are the data introduced and summarized in Ref. [12].
- Figure 3. Electromagnetic contribution to ρ^0-ω mixing
- Figure 4. ρ^0-ω exchange contributions
  (a) Short range, strong interaction effect;
  (b) Long range, electromagnetic effect
- Figure 5. Quark model of ρ^0-ω mixing
- Figure 6. Model of Krein & Thomas - q^2 variation of $< \rho^0|H_{str}|\omega >$
- Figure 7. Quark model of the ρ^0-γ^* transition.
- Figure 8. q^2 variation of $\frac{1}{f_\rho}$. The magnitude of f_ρ has been scaled to allow a comparison with the q^2 dependence of $< \rho^0|H_{str}|\omega >$.
- Figure 9. CSB in np scattering. This is after Fig. 3 of Ref. [24].

Table 1. Calculations of Δa_{CD}

	Henley, Morrison [1] 1966	Ericson, Miller [7] 1983	Cheung, Machleidt [9] 1986
OPEP	3.5	3.5 ± 0.2	3.8 ± 0.2^a
TPEP (all)	0.90	0.88 ± 0.1	0.8 ± 0.1
Coupling Constants	b	0^c	
$\gamma\pi$		1.1 ± 0.4^d	1.1 ± 0.4^d
Total		5.5 ± 0.3	5.7 ± 0.5

All values of Δa are in fm.

a. This also includes the effects of $\pi\rho, \pi\omega$ and $\pi\sigma$ exchanges

b. HM showed that one could choose charge dependent coupling constants to describe the remainder of Δa_{CD}, but these were unknown

c. The effect of using charge dependent coupling constants tends to cancel if these are used consistently in OPEP and TPEP

d. This is an average [7] of the results of Refs. [9] and [10]

Table 2. Blunden Iqbal calculation (see text)

A	orbit	Required CSB (keV)		Calc. CSB (keV)	
		DME	SkII	total	$\rho^0\omega$
15	$p_{3/2}^{-1}$	250	190	210	182
	$p_{1/2}^{-1}$	380	290	283	227
17	$d_{5/2}$	300	190	144	131
	$1s_{1/2}$	320	210	254	218
	$d_{3/2}$	370	270	246	192
39	$1s_{1/2}^{-1}$	370	270	337	290
	$d_{3/2}^{-1}$	540	430	352	281
41	$f_{7/2}$	440	350	193	175
	$1p_{3/2}$	380	340	295	258
	$1p_{1/2}$	410	330	336	282

The DME and SkII calculations are from Ref. 31.

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